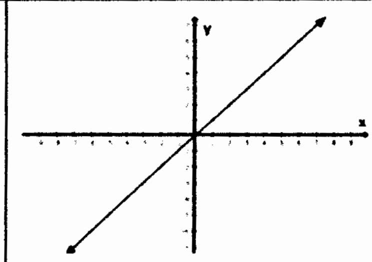
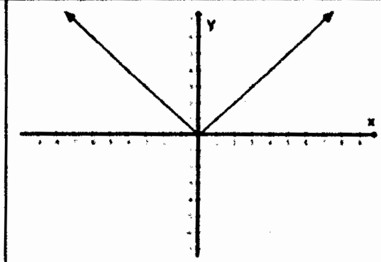
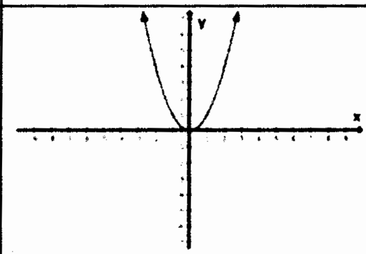
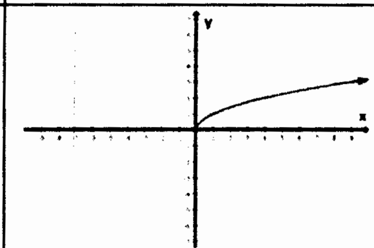
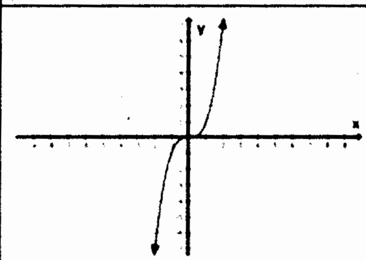
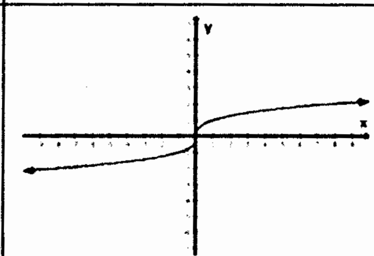
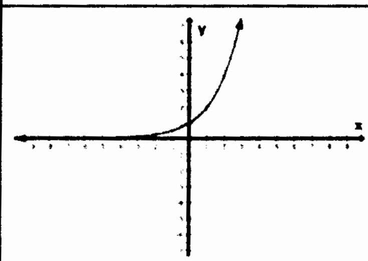
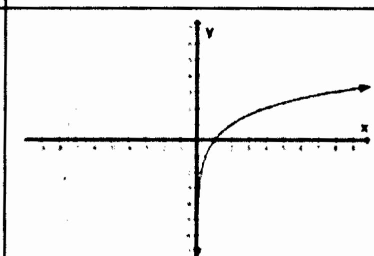
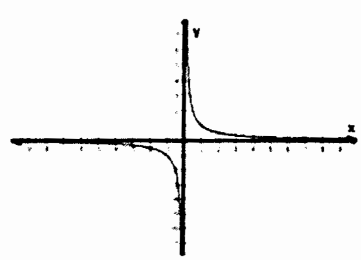
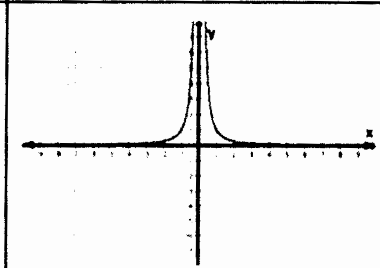
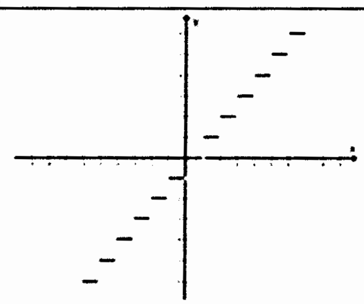
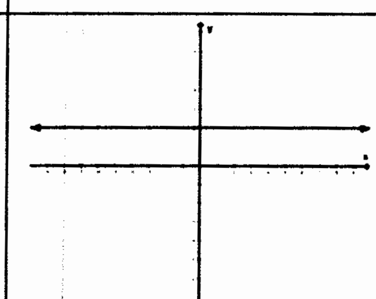
Math 250 1.2 Representing Functions

Objectives:

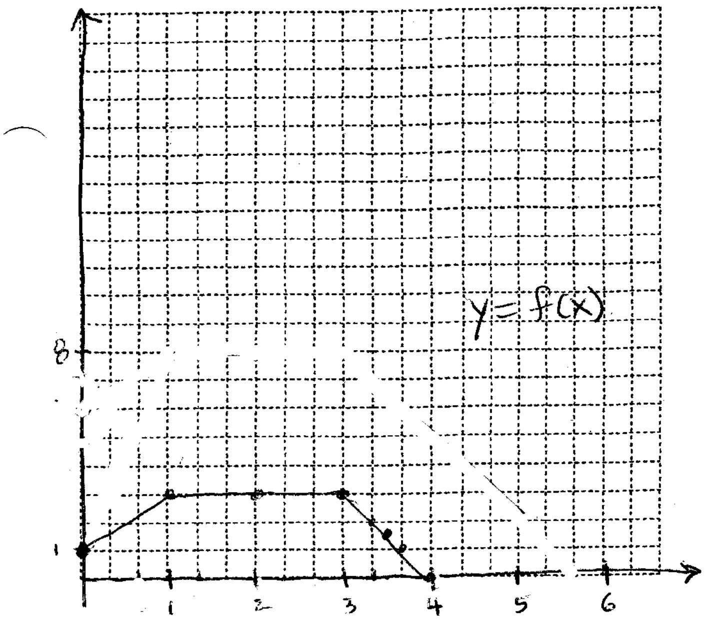
1) Classify functions: polynomials, rational, radical, algebraic, or transcendental.
   a. **Polynomials** are functions of the form $p(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$, where $\{a_i\}$ are constants, called **coefficients**, n is a positive integer called the **degree** of the polynomial, and up to n numbers $\{r_i\}$ where $p(r_i) = 0$ are called the **roots, zeros,** or x-intercepts of the polynomial.
   b. A **rational** function is a function of the form $f(x) = \dfrac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials.
   c. **Algebraic** functions are finite combinations of add, subtract, multiply, divide, rational exponents.
   d. Functions which are not algebraic are **transcendental,** for example, trig, logs, natural exponentials.
2) Sketch the graph of a function – Both analytic methods and technology are usually needed.
   a. Find all intercepts of a graph (x-int: set $y = 0$ and solve for x) (y-int: set $x = 0$ and solve for y)
   b. Find the domain and range.
   c. Identify any vertical or horizontal asymptotes.
   d. Use leading coefficient test for polynomials. ($a > 0$ always up on the right)
   e. Use transformations of a "parent function"
   f. Use symmetry, (determine if function is odd, even or neither)
   g. Identify local maximum values ("peaks") and local minimum values ("valleys")
   h. Use GC to check by generating tables or using zoom in or zoom out
3) Piecewise functions
   a. Graph piecewise functions (The result always passes the VLT! It's a function.)
   b. Given a graph, write a piecewise function describing it.
4) Given a function $f(x)$ or its graph, find the slope function $g(x)$ corresponding to it.
5) Given a function $f(x)$ or its graph, find the area function $A(x)$ corresponding to it.

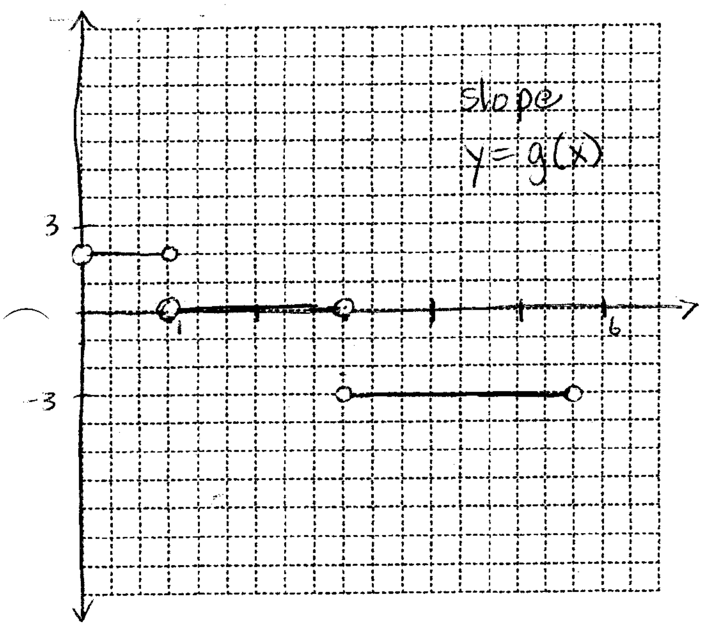
Practice

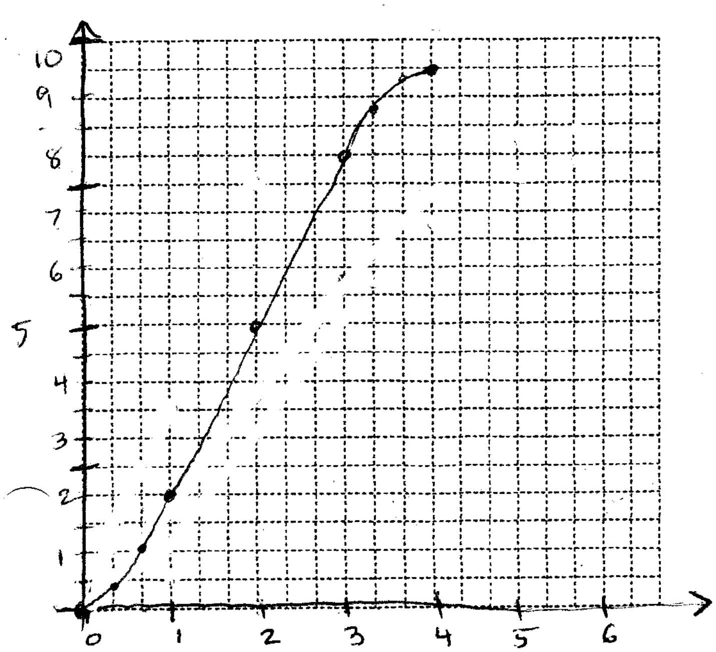
1) Evaluate, determine domain and range, and graph, using $f(x) = \begin{cases} 2x+1 & 0 \le x \le 1 \\ 3 & 1 < x \le 3 \\ -3x+12 & 3 < x \le 4 \end{cases}$
   a. $f(0)$
   b. $f(3.5)$
   c. $f(2)$
   d. Graph $f(x)$
   e. Find domain.
   f. Find range.
   g. Write $g(x)$ as a piecewise function.
   h. Graph the slope function $g(x)$
   i. Graph the area function $A(x)$
   j. Bonus: Write the area function $A(x)$ as a piecewise function.
2) Evaluate, determine domain and range, and graph, using $f(t) = |t-2|+1$, $0 \le t \le 20$
   a. $f(0)$
   b. $f(1)$
   c. $f(6)$
   d. Graph $f(x)$
   e. Find domain.
   f. Find range.
   g. Graph the slope function $g(x)$
   h. Write $g(x)$ as a piecewise function.
   i. Graph the area function $A(x)$
3) Identify the transformation from $y = f(x)$. Write each function as a composition, and define each component function.
   a. $y = f(x) - 5$
   b. $y = -f(x-4)$
   c. $y = f(x-c)+b$

Parent Function	Graph	Parent Function	Graph
$y=x$ Linear Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$		$y=\lvert x\rvert$ Absolute Value Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$	
$y=x^2$ Quadratic Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$		$y=\sqrt{x}$ Square Root Neither Domain: $[0,\infty)$ Range: $[0,\infty)$	
$y=x^3$ Cubic Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$		$y=\sqrt[3]{x}$ Cubic Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$	
$y=b^x,\ b>1$ Exponential Neither Domain: $(-\infty,\infty)$ Range: $(0,\infty)$		$y=\log_b(x),\ b>1$ Log Neither Domain: $(0,\infty)$ Range: $(-\infty,\infty)$	
$y=\frac{1}{x}$ Rational or Inverse Odd Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(-\infty,0)\cup(0,\infty)$		$y=\frac{1}{x^2}$ Inverse Squared Even Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(0,\infty)$	
$y=\text{int}(x)=[x]$ Greatest Integer Neither Domain: $(-\infty,\infty)$ Range: $\{y: y\in\mathbb{Z}\}$ (only integers)		$y=C$ Constant Function Even Domain: $(-\infty,\infty)$ Range: $\{y: y=C\}$	

Math 250 1.2







Practice

① $f(x) = \begin{cases} 2x+1 & 0 \le x \le 1 \\ 3 & 1 < x \le 3 \\ -3x+12 & 3 < x \le 4 \end{cases}$

x	f(x)
0	1
1/3	$1.\overline{6}$
2/3	$2.\overline{3}$
1	3
2	3
3	3
4	0
3.5	1.5
$3\frac{1}{3}$	2
$3\frac{2}{3}$	1

a) $f(0)$

$x=0$ is in $0 \le x \le 1$

$f(0)$ uses $2x+1$

$f(0) = 2(0)+1 = \boxed{1}$

b) $f(3.5)$

$x=3.5$ is in $x>3$

$f(3.5) = -3(3.5)+12 = \boxed{1.5}$

c) $f(2)$

$x=2$ is in $1<x\le 3$

$f(2) = \boxed{3}$

d) first graph

e) domain $[0,4]$

f) range $[0,3]$

h) slope function = slope at x

g) $g(x) = \begin{cases} 2 & 0<x<1 \\ 0 & 1<x<3 \\ -3 & 3<x<4 \end{cases}$

Note that the slopes at the "corners" are not defined, and show as open circles

i) area function

$A(q)$ = area between graph of $f(x)$, y-axis, x-axis, and the vertical line $x=q$

$A(0) = 0$ (no area yet)

$A(1)$ = area of trapezoid

$A = \frac{1}{2}(b_1+b_2)h$

$= \frac{1}{2}(1+3)\cdot 1 = 2$

$1 = b_1$, $b_2 = 3$, $h=1$

$A(2)$ = area $A(1)$ + rectangle from $[1,2]$

$= 2 + (1)(3) = 5$

$A(3)$ = area $A(2)$ + rect $[2,3]$

$= 5 + (1)(3) = 8$

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$A(4) = A(3) + \underset{[3,4]}{\text{triangle}}$

$= 8 + 1.5$

$= 9.5$

$H=3$

$B=1$

$\frac{1}{2}B \cdot H$

$= \frac{1}{2}(1)(3)$

$= \frac{3}{2}$

$A\left(\frac{1}{3}\right) = \frac{4}{9} = .\overline{4}$

$A\left(\frac{2}{3}\right) = 1.\overline{1} = \frac{10}{9}$

$A\left(\frac{10}{3}\right) = 8.8\overline{3} = \frac{53}{6} = 8\frac{5}{6}$

$A\left(\frac{11}{3}\right) = 9.\overline{3} = \frac{28}{3} = 9\frac{1}{3}$

To do on GC:

$y_1 \boxed{=} (2x+1) * y_4$

$y_2 \boxed{=} (3) * y_5$

$y_3 \boxed{=} (-3x+12) * y_6$

$\left.\begin{array}{l} y_4 = x \geq 0 \text{ and } x \leq 1 \\ y_5 = x \geq 1 \text{ and } x \leq 3 \\ y_6 = x \geq 3 \text{ and } x \leq 4 \end{array}\right\}$ turn off graphing

To insert y_4,
- $\boxed{\text{VARS}}$
- $\boxed{\triangleright}$ to y-vars
- 1. Function $\boxed{\text{enter}}$
- 4. Y_4 $\boxed{\text{enter}}$

ditto Y_5 & Y_6

fnint$(y_1+y_2+y_3, x, 0, __)$

$\Uparrow$ value of x to be evaluated, above $\frac{1}{3}, \frac{2}{3}, \frac{10}{3}, \frac{11}{3}$

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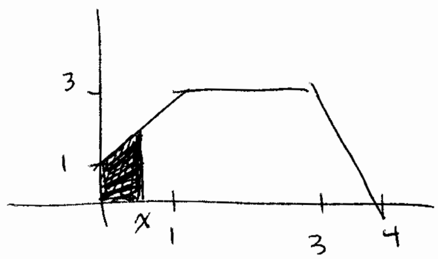
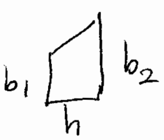
j) The area function $A(x)$ gives the area between the x-axis, the y-axis, the function $f(x)$, and a vertical line at x



For $x<1$, we want the area of the shaded trapezoid



$A = \frac{1}{2}(b_1+b_2)h$

$b_1 = 1$ (y-int of 1st piecewise line segment)

$b_2 = f(x) = 2x+1$ y-coord @ x

$h = x - 0 = x$

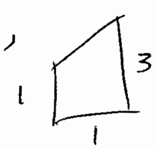
$$\begin{aligned} A(x) &= \tfrac{1}{2}(1+2x+1)\cdot x \\ &= \tfrac{1}{2}(2x+2)\cdot x \\ &= (x+1)(x) \\ &= x^2+x. \end{aligned}$$

<u>if</u> $0<x<1$

check endpoints.

If $x=0$, no area. $A(0) = 0^2+0 = 0$ ✓

If $x=1$,

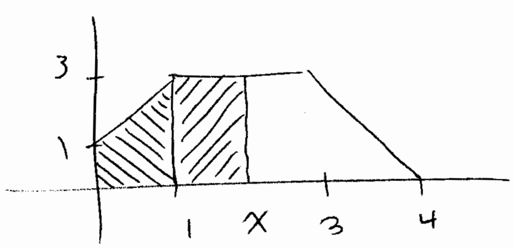


$\left.\begin{array}{l} A = \frac{1}{2}(1+3)(1) = 2 \\ A(1) = 1^2+1 = 2 \end{array}\right\}$ ✓

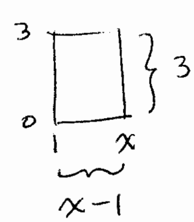
So we can include endpoints 0 & 1

$A(x) = x^2+x$ if $0 \le x \le 1$

Repeat for next piece: $1<x<3$



shaded area = trapezoid + rectangle



$$\begin{aligned} A(x) &= 2 + B\cdot H \\ &= 2 + 3(x-1) \\ &= 2+3x-3 \\ &= 3x-1. \end{aligned}$$

check endpoints.

If $x=3$, $B=2 \Rightarrow$ $\left.\begin{array}{l} 2+3(2)=8 \\ A(3) = 3(3)-1 = 8 \end{array}\right\}$ ✓

If $x=1$, $\left.\begin{array}{l} \text{Area} = 2 \\ A(1) = 3(1)-1 = 2 \end{array}\right\}$ ✓

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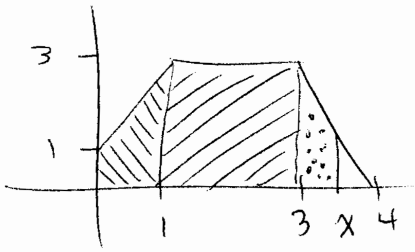
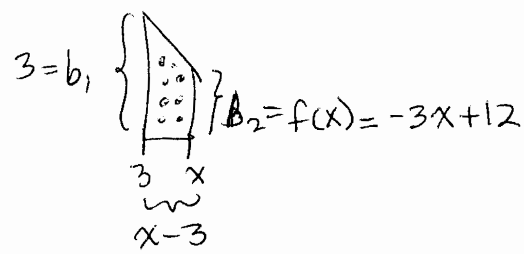
So the first two pieces of $A(x) = \begin{cases} x^2 + x & 0 \le x \le 1 \\ 3x - 1 & 1 \le x \le 3 \end{cases}$

Repeat for third piece: $3 < x < 4$



shaded area = trapezoid + rectangle + trapezoid

$= 2 + 6 + \frac{1}{2}(b_1 + b_2)h$



$= 8 + \frac{1}{2}(3 + -3x + 12)(x-3)$

$= 8 + \frac{1}{2}(-3x + 15)(x-3)$

$= 8 - \frac{3}{2}(x-5)(x-3)$

$= 8 - \frac{3}{2}(x^2 - 8x + 15)$

$= 8 - \frac{3}{2}x^2 + 12x - \frac{45}{2}$

$= -\frac{3}{2}x^2 + 12x - \frac{29}{2}$

check endpoints

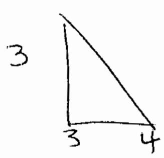
$A(3) = 8 \qquad -\frac{3}{2}(3)^2 + 12(3) - \frac{29}{2} = 8$ ✓

when $x=4$

$A = \frac{1}{2}B \cdot H$

$= \frac{1}{2}(1)(3)$

$= \frac{3}{2}$



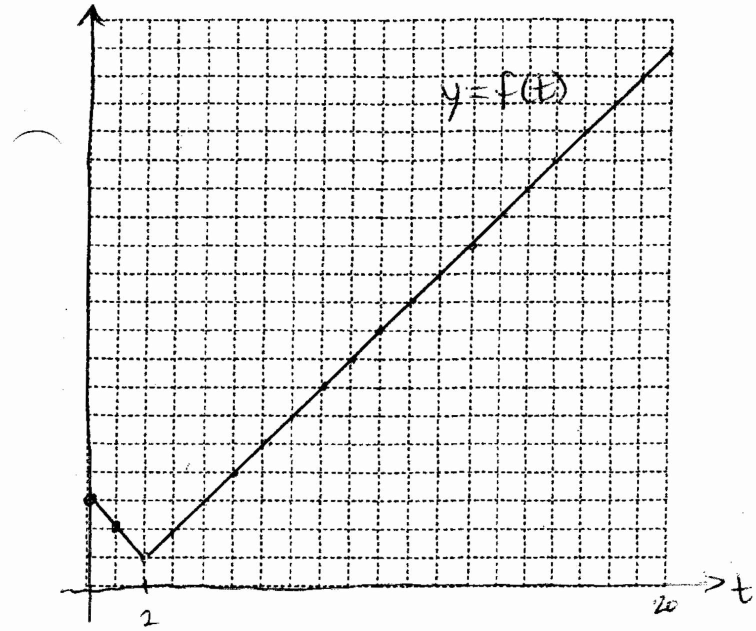
$A(4) = 2 + 6 + \frac{3}{2} = 9.5$ by geometry

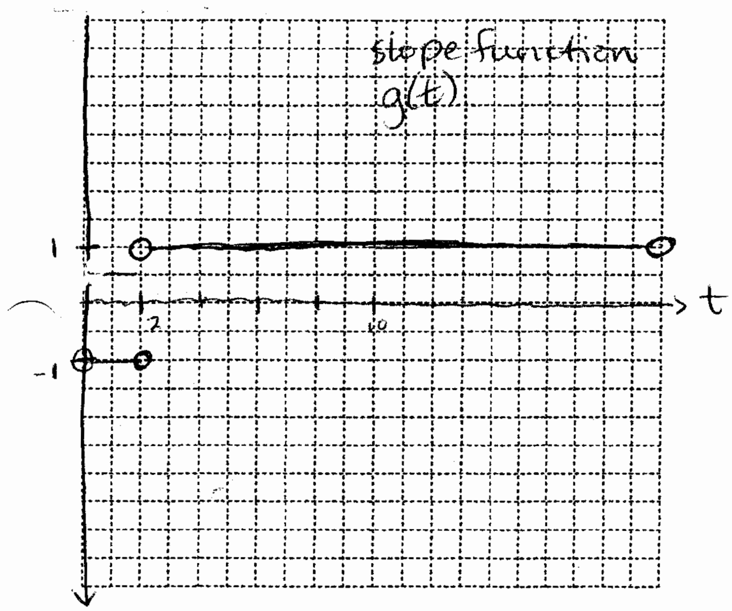
$A(4) = -\frac{3}{2}(4)^2 + 12(4) - \frac{29}{2} = 9.5$ by formula ✓

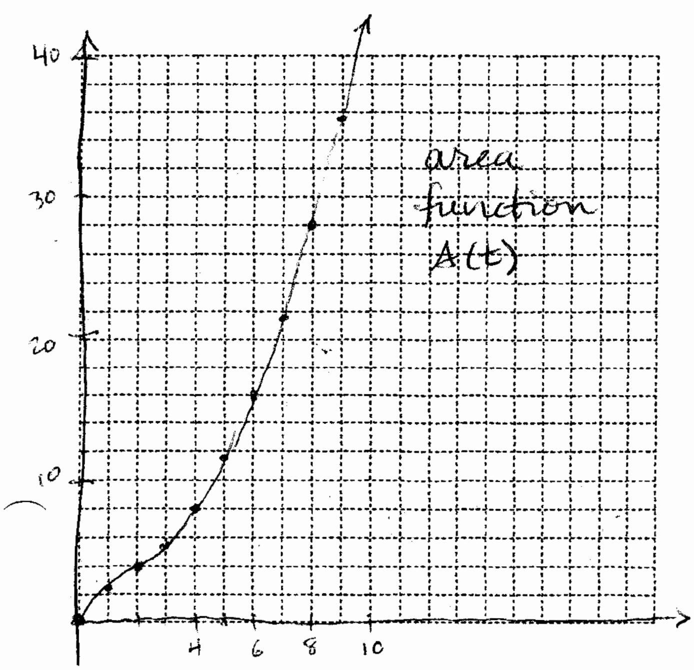
So area function

$$A(x) = \begin{cases} x^2 + x & 0 \le x \le 1 \\ 3x - 1 & 1 \le x \le 3 \\ -\frac{3}{2}x^2 + 12x - \frac{29}{2} & 3 \le x \le 4 \end{cases}$$

Math 250 1.2







Practice

② $f(t) = |t-2| + 1 \quad 0 \le t \le 20$

a) $f(0) = |0-2|+1$
$= |-2|+1$
$= 2+1$
$= \boxed{3}$

b) $f(1) = |1-2|+1$
$= |-1|+1$
$= 1+1$
$= \boxed{2}$

c) $f(6) = |6-2|+1$
$= |4|+1$
$= 4+1$
$= \boxed{5}$

t	$f(t)$
0	3
1	2
2	1
3	2
4	3
5	4
6	5
7	6

d) graph

e) domain $[0,20]$

f) range $[1,19]$

g) slope function

h) $g(t) = \begin{cases} -1 & t < 2 \\ 1 & t > 2 \end{cases}$

i) Area function

$A(0) = 0$

$A(1) = \underset{[0,1]}{\text{trapezoid}} = \frac{1}{2}(3+2)(1) = \frac{5}{2} = 2.5$

$A(2) = A(1) + \underset{[1,2]}{\text{trap}} = 2.5 + \frac{1}{2}(2+1)(1) = 4$

$A(3) = A(2) + \underset{[2,3]}{\text{trap}} = 4 + \frac{1}{2}(2+1)(1) = 5.5$

$A(4) = A(3) + \underset{[3,4]}{\text{trap}} = 5.5 + \frac{1}{2}(2+3)(1) = 8$

$A(5) = A(4) + \underset{[4,5]}{\text{trap}} = 8 + \frac{1}{2}(3+4)(1) = 11.5$

$A(6) = A(5) + \underset{[5,6]}{\text{trap}} = 11.5 + \frac{1}{2}(4+5)(1) = 16$

$A(7) = A(6) + \underset{[6,7]}{\text{trap}} = 16 + \frac{1}{2}(5+6)(1) = 21.5$

$A(8) = A(7) + \underset{[7,8]}{\text{trap}} = 21.5 + \frac{1}{2}(6+7)(1) = 28$

$A(9) = 35.5$

$A(10) = 44$

$A(15) = 101.5$

$A(20) = 184$

M250

3 Identify transformation of $y = f(x)$.
Write as composition.

a) $y = f(x) - 5$

vertical translation, down 5 units

$g(x) = x - 5$

$y = g(f(x))$

note:
$f(g(x)) = f(x-5)$

b) $y = -f(x-4)$

reflection over x-axis
horizontal translation, right 4 units

$g(x) = x - 4$

$h(x) = -x$

$y = h(f(g(x)))$

c) $y = f(x-c) + b$

horizontal translation c units — right if $c > 0$, left if $c < 0$

vertical translation b units — up if $b > 0$, down if $b < 0$

$g(x) = x - c$

$h(x) = x + b$

$y = h(f(g(x)))$